Math 53, Discussion 105

Wednesday, April 8, 2020 9:06 AM

Midterm material: 14.3 - 16.2

1. Chapter 14 (partial derivatives)
    a. Partial derivatives
    b. Tangent planes (linear approximation)
    c. Chain rule
    d. Directional derivatives and gradient vector
    e. Max and min values
2. Chapter 15 (Multiple Integrals)
    a. Double integrals over arbitrary domains
    b. Double integrals in polar coordinates
    c. Triple integrals over domains in:
        i. Rectangular coordinates
        ii. Cylindrical coordinates
    d. Change of variables (Jacobian)
3. Chapter 16 (vector calculus)
    a. Vector fields
    b. Line integrals

Cool math videos about covid

1) numberphile
2) 3blue1brown
3) Kurzgesagt
4) SIR model for pandemics

find $I = \int_{-\infty}^{\infty} e^{-x^2}\,dx$

$$Ex \quad I^2 = \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy} \longrightarrow \int_0^{2\pi}\int_0^{\infty} e^{-r^2} r\,dr\,d\theta$$

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}(y/x)$

A joint density function is given by

$$f(x,y) = \begin{cases} kx^3 & \text{for } 0 \le x \le 2 \text{ and } 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant k.

$k =$ 1/4

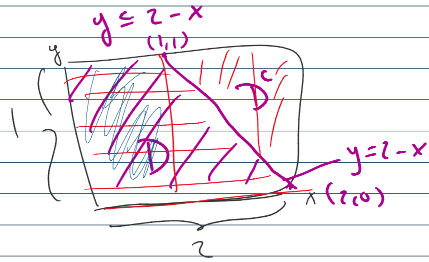
(b) Find the probability that (x, y) satisfies $x + y \le 2$.

probability = $= \iint_D f(x,y)\,dx\,dy$

find D

(c) Find the probability that (x, y) satisfies $x \le 1$ and $y \le 0.5$.

probability =



a) require $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1 = \underbrace{\int_0^1\int_0^2 kx^3\,dx}\,dy$

$\frac{kx^4}{4}\Big|_0^2 = 4k$

$\int_0^1 4k\,dy = 4k \Rightarrow \quad 4k = 1 \Rightarrow k = 1/4$

$x + y \le 2$

$\Rightarrow x \le 2 - y$

b) $1 - \iint_{D^c} f\,dx\,dy = 1 - \int_1^2\int_{2-x}^1 \frac{1}{4}x^3\,dy\,dx$

$\boxed{\iint_D f = \int_0^1\int_0^{2-y} f\,dx\,dy}$

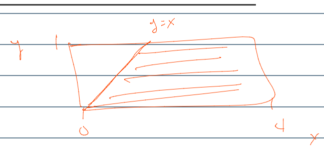
$= \underbrace{\int_0^1\int_0^1 f} + \int_1^2\int_0^{2-x} f\,dy\,dx$

Add Show path ... GLOBAL Usage: 153, Attempts: 5.27, Status: 100%; LOCAL Usage: 396, Attempts: 2.22, Status: 95%

Let p be the joint density function such that $p(x,y) = \frac{1}{4}xy$ in R, the rectangle $0 \le x \le 4, 0 \le y \le 1$, and $p(x,y) = 0$ outside R. Find the fraction of the population satisfying the constraint $x \ge y$

fraction =

$1 - \int_0^1\int_0^x \frac{1}{4}xy\,dy\,dx$

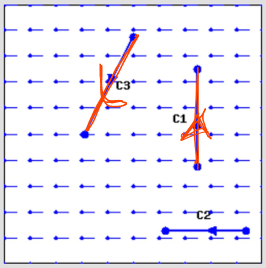


If C is the part of the circle $\left(\frac{x}{5}\right)^2+\left(\frac{y}{5}\right)^2=1$ in the first quadrant, find the following line integral with respect to arc length.

$\int_C (2x-3y)ds =$

Add Show path ... GLOBAL Usage: 238, Attempts: 3.05, Status: 99%; LOCAL Usage: 409, Attempts: 3, Status: 100%

Consider the vector field $\vec{F}$ shown in the figure below together with the paths C_1, C_2, and C_3.



(**Note:** *For the vector field, vectors are shown with a dot at the* ***tail*** *of the vector.*)

Arrange the line integrals $\int_{C_1}\vec{F}\cdot d\vec{r}$, $\int_{C_2}\vec{F}\cdot d\vec{r}$ and $\int_{C_3}\vec{F}\cdot d\vec{r}$ in ascending order:

? < ? < ?

$F(x,y) = \langle +1, 0 \rangle$

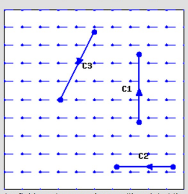
$\int_{C_2} \vec{F}\cdot d\vec{r} \;<\; \leq \int_{C_3} \;\leq\; \int_{C_1} F$

If C is the part of the circle $\left(\frac{x}{5}\right)^2+\left(\frac{y}{5}\right)^2=1$ in the first quadrant, find the following line integral with respect to arc length.

$\int_C (2x-3y)ds =$

Add Show path ... GLOBAL Usage: 238, Attempts: 3.05, Status: 99%; LOCAL Usage: 409, Attempts: 3, Status: 100%

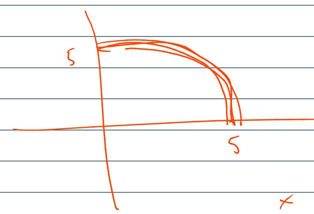
Consider the vector field $\vec{F}$ shown in the figure below together with the paths C_1, C_2, and C_3.



(**Note:** *For the vector field, vectors are shown with a dot at the* ***tail*** *of the vector.*)

Arrange the line integrals $\int_{C_1}\vec{F}\cdot d\vec{r}$, $\int_{C_2}\vec{F}\cdot d\vec{r}$ and $\int_{C_3}\vec{F}\cdot d\vec{r}$ in ascending order:

? < ? < ?



$\iint 5x^2 + 5y^2$

$x = r\cos\theta$

$y = r\sin\theta$

$\iint_0 5r^2$

$\iint r^2$

Parametrize $C := \vec{r}(t) = \langle \underbrace{5\cos(t)}_{x}, \underbrace{5\sin(t)}_{y} \rangle$

$0 \leq t \leq \pi/2$

$\sqrt{5^2\sin^2(t) + 5^2\cos^2(t)} = 5$

$$\iint_C f\, ds = \int_a^b f(r(t)) \sqrt{x'^2 + y'^2}\, dt$$

$$= \int_0^{\pi/2} \underbrace{(2\cdot 5\cos(t) - 3\cdot 5\sin(t))}_{f(r(t))} \cdot 5 \, dt$$

$$= 25 \int_0^{\pi/2} 2\cos(t) - 3\sin(t)\, dt$$

$$= 25\left(\underbrace{2\sin(t)\Big|_0^{\pi/2}}_{2} + \underbrace{3\cos(t)\Big|_0^{\pi/2}}_{-3} \right) = -25$$

$dx\,dy$

$\cos$

$\sin\theta$